

1 Aggregation of Experts Judgments for Climate
2 Tipping Points

3 Marcello Basili¹ and Federico Crudu^{1,2}

4 ¹Department of Economics and Statistics, University of Siena, Piazza
5 San Francesco 7, Siena, Italy.

6 ²Centre For North South Economic Research, Via S. Giorgio 12,
7 Cagliari, Italy.

8 Contributing authors: marcello.basili@unisi.it; federico.crudu@unisi.it;

9 **Abstract**

10 This paper introduces a method for the evaluation of the occurrence of tipping
11 points based on the combination of probability intervals from experts judgments
12 elicited in face-to-face interviews. The computation of such conditional probab-
13 ilities is based on the aggregation of imprecise probability judgments through the
14 Steiner point. The probability of a tipping point can be updated by the standard
15 Bayes rule to generate tipping point scenarios. Our results suggest that tipping
16 events may happen with relatively large probabilities, in contrast with the view
17 that tipping points are low-probability-high-impact events. This suggests that
18 mitigation and containment policies cannot be further postponed.

19 **Keywords:** Bayesian updating; aggregation of opinions; global warming; judgmental
20 forecasting; Steiner point; tipping points

21 **JEL Classification:** Q54; D81; C10

1 Introduction

As of today, there is a large literature suggesting that the consequences of climate change may involve abrupt changes and tipping points (e.g. [Lenton et al., 2019](#)).¹ [Armstrong McKay et al. \(2022\)](#) identify a finite core of tipping point elements able to modify the Earth system functioning and show that six climate tipping points global elements are likely even in the Paris Agreement range of 1.5°C to 2°C warming. The concept of tipping point was introduced into the scientific debate in the '80s of the last century by the IPCC to represent large scale discontinuities in the climate system. At that time, experts believed that tipping points would be crossed if global warming had exceeded 5°C. In the 2018 IPCC report ([IPCC, 2018b](#)) experts suggested that tipping points could be crossed even between 1 and 2°C of global warming.²

The main objective of this paper is to introduce a new methodology for the evaluation of occurrence of tipping points based on the combination of probability intervals from experts judgments elicited in face-to-face interviews. When experts face uncertainty or have imprecise knowledge about future states of the world, alternative approaches that differ from Bayesian pool methods are required. In this paper we apply an aggregation method based on the Steiner point introduced in [Basili and Chateauneuf \(2020\)](#). This method assumes that experts have imprecise information represented by convex sets of probability distributions with the requirement that the intersection of these sets is not empty and experts have at least one common probability distribution. A desirable feature of this approach is that the opinion pooling is the probability distribution of the tipping points we considered. This differs from other methods where some type of (simple or weighted) average is defined (see, e.g., [Clemen & Winkler, 1999, 2007](#)). Furthermore, by means of a standard Bayesian updating rule

¹A tipping point is defined as “[a] level of change in system properties beyond which a system reorganizes, often abruptly, and does not return to the initial state even if the drivers of the change are abated. For the climate system, it refers to a critical threshold when global or regional climate changes from one stable state to another stable state” ([IPCC, 2018b](#)). See also the recent article by [Armstrong McKay et al. \(2022\)](#) for a review and a thorough definition of tipping point.

²[Lenton et al.](#) mention that some “models suggest that the Greenland Ice sheet could be doomed at 1.5°C of warming, which could happen as soon as 2030” ([Lenton et al., 2019](#), p. 592).

we can provide posterior probabilities that may be used to generate future scenarios or hypotheses for future studies, once more data become available (see Section 3 for more details on the aggregation approach).³ It is important to notice that the construction of the Steiner point requires that the intersection of the experts' opinion be non empty. However, if that were not the case, the aggregation could be applied to subsets of consistent experts (see Section 3.1).

To provide an assessment of the occurrence of tipping points we use the elicitation data in Kriegler et al. (2009).⁴ In Kriegler et al.'s seminal paper, experts are asked to provide probability intervals about three temperature scenarios. In this context, intervals of probabilities are a representation of uncertain and imprecise judgements.⁵ The aggregation method proposed in this paper is new in the context of problems related to climate change.

The paper proceeds as follows. Section 2 briefly discusses the related literature. Section 3 introduces the fundamental aspects of our theoretical framework. In Section 4 we present the conditional probabilities of occurrence of various tipping points computed via the Steiner point as well as the results of the Bayesian updating. Finally, Section 5 concludes the study.

2 Related Literature

Communicating uncertainty about climate change is crucial to effectively influence policy decisions and shape public opinion. In this context, the IPCC special report Global Warming of 1.5°C (IPCC, 2018a) is an important example. The IPCC special

³It seems that there are no follow up studies on experts' assessment of tipping points. This is also confirmed in the correspondence with leading researchers in the field. Other recent studies use the data and results in Kriegler, Hall, Held, Dawson, and Schellnhuber (2009), see, e.g., Gaucherel and Moron (2017); Wunderling, Donges, Kurths, and Winkelmann (2021).

⁴The data are collected from Kriegler et al. (2009) supplemental appendix and are available at this link along with the replication code.

⁵In Kriegler et al. (2009) the low temperature scenario (*Low*) considers an increase between about 1°C and 2°C by year 2200 in comparison with year 2000. The medium temperature scenario (*Medium*) considers an increase between about 2°C and 4°C by year 2200 in comparison with year 2000. The high temperature scenario (*High*) considers an increase between about 4°C and 8°C by year 2200 in comparison with year 2000.

67 report is based “on the assessment of around 6,000 peer-review publications, most
68 of them published in the last few years” (IPCC, 2018a, p. v). The report aggregates
69 multiple forms of knowledge to address and communicate the degree of certainty (or
70 lack thereof) of specific findings.

71 In general, however, some critical issues emerge in the treatment of uncertainty:
72 findings are based on multiple lines of evidence and are expressed using confidence
73 qualifiers and many of them depend on certain model assumptions. In addition to
74 that, findings have to be updated if new information becomes available. Borsuk and
75 Tomassini (2005), Tomassini, Reichert, Knutti, Stocker, and Borsuk (2007), Knutti et
76 al. (2008), Zickfeld et al. (2007), Zickfeld, Morgan, Frame, and Keith (2010), Kriegler
77 et al. (2009) highlight that aggregation of probabilistic projections with a variety of
78 statistical models is an unsolved problem. One fundamental challenge of the assessment
79 process is to summarize such information into a unique quantity. However, due to the
80 heterogeneity in sources and quality of the information, obtaining a unique synthetic
81 measure may become a daunting task.

82 When formal statistical procedures are unavailable, expert judgment approaches
83 are often employed to provide an assessment of uncertainty (see e.g. Mastrandrea et
84 al., 2011).⁶ Significantly, “one option is to resort to imprecise probability (e.g., Kriegler
85 and Held (2005); Hall, Fu, and Lawry (2007); Tomassini et al. (2007)), that is, consider
86 an uncertainty in PDFs [probability distribution functions] or sets of PDFs” (Knutti
87 et al., 2008, p. 2658). If each expert has a set of probability distributions, the mean
88 value for each scenario is evaluated along with all the individual PDFs (Tomassini et
89 al., 2007).

90 Experts’ quantitative judgments are often elicited in face-to-face interviews. Then,
91 using a range of different procedures, mean (averaged over experts) ranks are computed
92 (Kriegler et al., 2009; Zickfeld et al., 2010). In Zickfeld et al. (2010), 14 experts (leading

⁶In 1975, the U.S. Nuclear Regulatory Commission (NRC) introduced for the first time a procedure for the elicitation process and, since then, techniques and methods have spread to other areas such as volcanology, public health, ecology, aeronautics, climatology etc. (Cooke, 2013).

93 climate scientists) discuss about three scenarios (high, medium, low) of net radiative
94 forcing at the top of the atmosphere from anthropogenic sources through the year
95 2200. Experts use a cardinal scale from 0 (no chance) to 1 (definite chance) for each
96 of the three forcing trajectories. Experts elicit probabilities for each scenario and in
97 the following step they are asked to estimate the median of the mean trajectories of
98 warming between 2000 and 2050. What they find is that it falls between 0.16°C/decade
99 and 0.36°C/decade.

100 Providing a realistic assessment of the probability of a tipping event may have a
101 fundamental impact in influencing policy decisions concerning climate change. Unfor-
102 tunately, there are very few models able to simulate abrupt climate changes as
103 consequences of realistic external forcing and, even when sophisticated climate mod-
104 els show effects, in particular a temperature response, they are weak with respect to
105 Dansgaard-Oeschger events.⁷ To manage the complexity and the uncertainty implied
106 in climate models, policy makers (PMs) have no alternative but to resort to experts
107 and obtain experts' elicitations; that is, expert probabilistic judgments (Colson &
108 Cooke, 2018), that are represented by imprecise probabilities or intervals of probab-
109 ities, generally. Once the elicitation is completed, the data are aggregated. Using a
110 reliable aggregation method to provide a synthesis of the various opinions seems of
111 paramount importance, also given the fact that no clear dominant approach exists.⁸

⁷Dansgaard-Oeschger events are rapid climate fluctuations such as the Greenland ice melting occurred in the Eemian interglacial. This event took the form of rapid warming episodes, followed by gradual cooling periods, that increased the average annual temperature on the Greenland ice sheet of 8 °C over 40 years (Dansgaard et al., 1993; Heinrich, 1988).

⁸See Lam and Majszak (2022) for a recent review on the challenges and opportunities of expert judgment for the assessment of climate tipping points.

112 3 Theoretical Framework

113 The aggregation process of probabilistic opinions (opinion pooling) entails a function,
114 known as a pooling rule, that elicits a consensus distribution. Such a consensus distri-
115 bution is determined among not necessarily independent and fully competent experts
116 when each of them has multiple priors on future states of the world.

117 Under uncertainty or deep uncertainty experts have partial, incomplete or fuzzy
118 knowledge and their beliefs cannot be represented by a unique, additive and fully
119 reliable probability distribution, but either by a finite set of them, an interval of
120 probabilities or by a non necessarily additive measure (e.g. a capacity) (e.g. [Basili &
121 Chateauneuf, 2011, 2016; Basili & Pratelli, 2015](#), and references therein).⁹

122 In this paper we assume that opinions are expressed through different probabil-
123 ity distributions and that there exists a PM who adopts a multiple priors decision
124 model. The set of probability distributions of all experts can be considered a reflection
125 of the PM’s assessment of the reliability of available information about the underly-
126 ing uncertainty, that is, her perception of uncertainty; the elicited aggregation rule
127 incorporates the PM’s attitude about scanty and vague information. Facing the set
128 of all probability distributions attached by experts to possible events, the PM evalu-
129 ates their probability intersection. Such a common opinion is the Steiner point of the
130 convex capacity that emerges from the aggregation of experts’ opinions.

131 The aggregation via the Steiner point provides a natural and tractable way to
132 summarize opinions and only assumes that experts have imprecise information but
133 nothing about their competence, experience and independence. This is, for example,
134 the case of non Bayesian groups, i.e. groups where experts do not have a unique and

⁹Deep uncertainty includes “situations in which we are still able (or assume) to bound the future around many possible plausible futures and situations in which we only know that we do not know” ([Marchau, Walker, Bloemen, & Popper, 2019](#)). See also [Rohmer, Le Cozannet, and Manceau \(2019\)](#) and [Frederikse et al. \(2020\)](#) for some recent discussion of the concepts of uncertainty in the context of climate change issues.

additive probability distribution.¹⁰ In addition, the Steiner point not only allows us to derive the consensus distribution about a given event, but it is amenable to Bayesian updating. This means that the PM can simply update the elicited distribution when new information is made available without calling for a new round of interviews. Whenever experts have opinions that are too heterogeneous the aggregation cannot be performed (see the G-consistency condition below in this section). Yet, the Steiner point can be applied to subsets of individuals allowing us to identify different trends and common opinions among consistent experts. In general, the aggregation through the Steiner point suggests that the experts are consistent and that the interpretation of a tipping point as a high-impact-low-probability event seems to be incorrect (see also [Lenton & Ciscar, 2013](#); [Lenton et al., 2019](#)). We find, in fact, that the probability of crossing a tipping point threshold is non trivially larger than zero in nearly all cases, even for low climate change scenarios. Furthermore, it has been claimed ([Lenton & Ciscar, 2013](#)) that a more adequate picture for the representation of tipping points would be to provide the (joint) probability distribution for tipping each element.

3.1 Experts' Consistency

Let us consider a finite set $\mathcal{S} = \{s_1, \dots, s_n\}$ of states of the world and let $\Sigma = 2^{\mathcal{S}}$ be the σ -algebra associated to the set \mathcal{S} . Since the study deals with discrete events, P is a probability mass function (pmf) on (\mathcal{S}, Σ) such that $P : \Sigma \rightarrow [0, 1]$. For a specific pmf, $p(s_i) = P(S = s_i)$ for any $i \in \{1, \dots, n\}$ and $\sum_{i=1}^n p(s_i) = 1$.

Consider now a non-negative set function $v : \Sigma \rightarrow \mathbb{R}$. Such a function is a capacity (a non additive probability) on (\mathcal{S}, Σ) if, for any $\mathcal{A}, \mathcal{B} \in \Sigma$, $\mathcal{A} \subseteq \mathcal{B} \Rightarrow v(\mathcal{A}) \leq v(\mathcal{B})$, $v(\emptyset) = 0$ and $v(\mathcal{S}) = 1$, where \emptyset is the empty set. The capacity $v(\cdot)$ is convex if

¹⁰The experts that provide judgements for the IPCC assessment may be interpreted as a non Bayesian group, as they typically provide evaluations in terms of qualitative levels of confidence and quantitative likelihoods of occurrence of a given event generally expressed as intervals of probabilities. [Mastrandrea et al. \(2011\)](#) characterize the degree of certainty in key findings with two metrics: “[c]onfidence in the validity of a finding, based on the type, amount, quality, and consistency of evidence (e.g., mechanistic understanding, theory, data, models, expert judgment) and the degree of agreement, [c]onfidence is expressed qualitatively” and “[q]uantified measures of uncertainty in a finding expressed probabilistically (based on statistical analysis of observations or model results, or expert judgment)”.

158 $v(\mathcal{A} \cup \mathcal{B}) + v(\mathcal{A} \cap \mathcal{B}) \geq v(\mathcal{A}) + v(\mathcal{B})$. The corresponding dual capacity is defined as
 159 $\bar{v}(\mathcal{A}) = 1 - v(\mathcal{A}^c)$. The dual capacity $\bar{v}(\mathcal{A})$ is concave. Uncertainty is modeled via the
 160 core of the convex capacity v . The core $\mathcal{C}(v)$ is a set of probability distributions P on
 161 (\mathcal{S}, Σ) such that $P(\mathcal{A}) \geq v(\mathcal{A}) \forall \mathcal{A} \in \Sigma$.

162 The PM's problem is to learn the distribution of a certain event by aggregating
 163 experts' opinions. Let P_0 denote the unobserved pmf that governs the phenomenon
 164 under study. The PM asks expert j , ($j = 1, \dots, m$), to provide a lower and upper
 165 bound for the probability $p_i = p_0(s_i) = P_0(S = s_i)$. The set of possible probabilities
 166 considered by expert j is defined as

$$\mathcal{P}^j = \left\{ P^j = (p_1^j, \dots, p_i^j, \dots, p_n^j) : a_i^j \leq p_i^j \leq b_i^j, i = 1, \dots, n \right\}.$$

167 The PM will accept the expert's opinion if two consistency conditions are met. Specif-
 168 ically, for the expert's opinions to make sense the bounds a_i^j and b_i^j must meet the
 169 following conditions of *individual consistency* (*I-consistency*).

170 **Condition 1. [I-consistency]** The bounds a_i^j and b_i^j satisfy the conditions $0 \leq$
 171 $a_i^j \leq b_i^j \leq 1$ and $\sum_{i=1}^n a_i^j \leq 1 \leq \sum_{i=1}^n b_i^j$.

172 The set \mathcal{P}^j is not empty if and only if the I-consistency condition is met.
 173 Furthermore, \mathcal{P}^j can be seen as the core $\mathcal{C}(v^j)$ of a given convex capacity v^j where

$$v^j(\mathcal{A}) = \max \left(\sum_{i \in \{i: s_i \in \mathcal{A}\}} a_i^j, 1 - \sum_{i \in \{i: s_i \notin \mathcal{A}\}} b_i^j \right) \quad (1)$$

174 and the set $\mathcal{A} \subseteq \mathcal{S}$ is a set of states of the world (Chateauneuf & Cornet, 2018;
 175 De Campos, Huete, & Moral, 1994). It is expected that the unknown distribution P_0
 176 be in the the set \mathcal{P}^j of the generic expert j , i.e. $P_0 \in \mathcal{P}^j$. In addition to that, it

177 is expected also that P_0 be in the intersection of all \mathcal{P}^j . Hence, the following *group*
 178 *consistency* (*G-consistency*) condition is supposed to be met.

179 **Condition 2. [G-consistency]** The intersection of the probability sets associated
 180 with the pool of experts is non empty, i.e., $\mathcal{P} = \cap_{j=1}^m \mathcal{P}^j \neq \emptyset$ and that $P_0 \in \mathcal{P}$.

181 Whenever the intersection set $\mathcal{P} = \emptyset$, this is, when experts have conflicting
 182 opinions, the PM may still be able to extract valuable information by applying the G-
 183 consistency condition to a subset of experts. This approach may reveal, for example,
 184 whether some experts are more or less optimistic about a given phenomenon.

185 3.2 Aggregation and Scenarios

186 The consensus distribution set is associated to a convex capacity $v(\cdot)$, defined as

$$v(\mathcal{A}) = \max \left(\sum_{i \in \{i: s_i \in \mathcal{A}\}} a_i, 1 - \sum_{i \in \{i: s_i \notin \mathcal{A}\}} b_i \right) \quad (2)$$

187 where $a_i = \max_j a_i^j$ and $b_i = \min_j b_i^j$. In this context the Steiner point is particularly
 188 relevant, since the Steiner point of $\mathcal{C}(v)$ is the center of the core of the convex capacity
 189 v and represents the consensus probability for the given set of experts. The Steiner
 190 point is denoted as $\Pi^v \in \mathcal{C}(v)$.

191 It is interesting to notice that when the set of states of the world is finite, the
 192 Steiner point coincides with the Shapley value (Basili & Chateauneuf, 2020; Pechersky,
 193 2015; Shapley, 1971) and it is easily computed via the following expression

$$\Pi_i^v = \sum_{s_i \in \mathcal{A} \subseteq \mathcal{S}} \frac{(|\mathcal{A}| - 1)!(n - |\mathcal{A}|)!}{n!} (v(\mathcal{A}) - v(\mathcal{A} \setminus \{s_i\})), \quad i = 1, \dots, n. \quad (3)$$

Equation (3) shows how the Shapley value represents the average marginal individual contribution over all the possible different permutations in which the grand coalition \mathcal{S} may be formed (Basili & Chateauneuf, 2020).¹¹

Our problem consists of two states of the world ($n = 2$), this is, whether a tipping point occurs (*Tip*) or it does not occur (*No Tip*). The two states of the world occur with, say, probability Π_1^v and $\Pi_2^v = 1 - \Pi_1^v$, respectively. Let us now define the random variables $T \in \{Tip, No\ Tip\}$ denoting the occurrence (or not) of a tipping point and $C \in \{Low, Medium, High\}$ representing possible temperature scenarios as defined in Section 1. Hence, we have the following Bayes rule

$$P(T = Tip|C) = \frac{P(C|T = Tip)P(T = Tip)}{P(C|T = No\ Tip)P(T = No\ Tip) + P(C|T = Tip)P(T = Tip)}. \quad (4)$$

The Bayes rule in equation 4 can be used to update the probability of occurrence of a tipping point obtained via the Steiner point. Specifically, we choose Π_1^v for $P(T = Tip)$ as a prior probability, while $P(C|T = Tip)$ and $P(C|T = No\ Tip)$ can be obtained from expert knowledge. Alternatively, we can feed the formula a grid of values. In this last case the posterior probability $P(T = Tip|C)$ can be graphically represented as a surface. For the empirical analysis in Section 4 we opt for the latter.

¹¹Recent research suggests that, for independent inputs, the Steiner point-Shapley value is bracketed between two different Sobol' indices. This result seems to hold also for the case of dependent inputs or expert judgments (Song, Nelson, & Staum, 2016, see also Owen and Prieur (2017) for further uses of the Shapley value in the context of ANOVA). Sobol' index provides a measure of the importance of inputs to a function and is defined in terms of the functional analysis variance decomposition (Sobol', 1990, 1993).

4 Eliciting Probabilities of Tipping Points by the Steiner Point

Kriegler et al. (2009) elicited beliefs of experts about the probability of triggering major changes in the Earth system associated to seven tipping points (Table 1 reports the tipping points under analysis) for three different global median temperature scenarios. As previously mentioned in Section 1, the three scenarios consider the crossing of a tipping point for a given temperature scenario and a fixed time horizon. Specifically, *Low* refers to an increase between about 1°C and 2°C by year 2200 in comparison with year 2000, *Medium* refers to an increase between about 2°C and 4°C by year 2200 in comparison with year 2000 and *High* refers to an increase between about 4°C and 8°C by year 2200 in comparison with year 2000. In light of the new evidence produced in over a decade, Armstrong McKay et al. (2022) classify new and known tipping elements as tipping points that act on a global scale or only locally (such as the dieback of Boreal forests, BOFO in Table 1). For other phenomena such as El Niño (NINO in Table 1), there seems to be insufficient evidence for it to be characterized by a tipping element. The decline of the ocean carbon sink (DOCS in Table 1), which was previously considered a tipping element, is categorized as threshold-free. For consistency of exposition with respect to Kriegler et al.’s data, we consider NINO and DOCS as if they were tipping elements.¹²

The raw data collected by Kriegler et al. (2009) are summarized in the dumbbell plots in Figures A1 to A7 in Appendix A. Each figure consists of three plots and each plot refers to one of the three temperature scenarios. To every row in the plots we attach one single expert. By direct inspection, we note, as also stressed in Kriegler et al. (2009), the tendency of the experts to place high probability in the high temperature scenarios.¹³

¹²Kriegler et al. (2009) report that BOFO and DOCS “were judged by experts to be of more speculative nature”

¹³The data are collected from Kriegler et al. (2009) supplemental appendix and are available at Federico Crudu’s personal webpage (link) along with the replication code.

AMAZ	Dieback of the Amazon rainforest.
BOFO	Dieback of Boreal forests.
AMOC	Reorganization of the Atlantic meridional overturning circulation.
DAIS	Disintegration of the West Antarctic ice sheet.
MGIS	Melt of the Greenland ice sheet.
DOCS	Decline in ocean carbon sink.
NINO	Shift to a more persistent El Niño regime.

Table 1 Tipping points considered in [Kriegler et al. \(2009\)](#).

In order to provide policy relevant information [Kriegler et al. \(2009\)](#) combine, using an aggregation rule, the probability intervals supplied by the experts for every tipping point and every temperature scenario.¹⁴ Their aggregation rule produces upper and lower probabilities. Our approach is substantially different. In fact, the Shapley value described in equation 3 returns the probability of occurrence of a tipping point conditional on a given temperature scenario.

Figure 1 features the probabilities associated to every tipping point and each climate scenario resulting from the aggregation of the experts' opinions via the application of equation 3. In Figure 1 we see that, in general, a high climate change scenario (red triangle) is more likely to trigger a tipping point. While this result is not too surprising, we also observe that there is a certain degree of heterogeneity across scenarios and tipping points. Specifically, AMOC has a very low chance to occur for the low climate change scenario. All the other tipping points have, for the same scenario a non negligible (say, larger than 10%) probability to occur. As we consider the higher climate change scenarios, *Medium* and *High*, the probability of having a tipping point increases. It is remarkable that for *High*, the probabilities of occurrence for BOFO, DAIS, DOCS and MGIS are very high.

To provide further intuition for the interpretation of the plots in Figure 1, we adapt the terminology used in [Kriegler et al. \(2009\)](#). This is, we label as *remote* the prospect of having a tipping point if $\Pi_1^v < 0.1$, *significant* if $\Pi_1^v \geq 0.1$ and *large* if

¹⁴[Kriegler et al. \(2009\)](#) introduce a novel aggregation rule called forced consensus pooling. See also [Clemen and Winkler \(1999\)](#) and [Nau \(2002\)](#).

254 $\Pi_1^v \geq 0.5$. Under these criteria only AMOC for the low temperature scenario has a
 255 remote chance of being triggered. DAIS and MGIS have a large probability of being
 256 triggered under the high temperature scenario, while BOFO and DOCS have a large
 257 probability of being triggered under both the medium and high temperature scenario.
 258 The remaining scenarios have a significant probability of triggering a tipping point.

259 By applying the Bayes rule in equation 4 we update the results obtained via
 260 the Steiner point to generate tipping point scenarios for different combinations of
 261 $P(C|T = No\ Tip)$ and $P(C|T = Tip)$. Every point in the resulting surfaces (figures 2
 262 to 8) is the conditional probability of crossing a tipping point given the likelihood of the
 263 (conditional) occurrence of the temperature scenario. We notice that the high temper-
 264 ature scenario produces large probabilities for the occurrence of tipping points. This
 265 result is particularly clear for BOFO (figure 3 (c)), AMOC (figure 4 (c)), DAIS (figure
 266 5 (c)), DOCS (figure 6 (c)) and MGIS (figure 7 (c)). Committing to low climate change
 267 scenarios may produce remote probabilities of tipping, at least for AMOC (figure 4
 268 (a)) and DAIS (figure 5 (a)), yet in most of the remaining cases the probabilities of
 269 triggering a tipping point are generally at least significant.

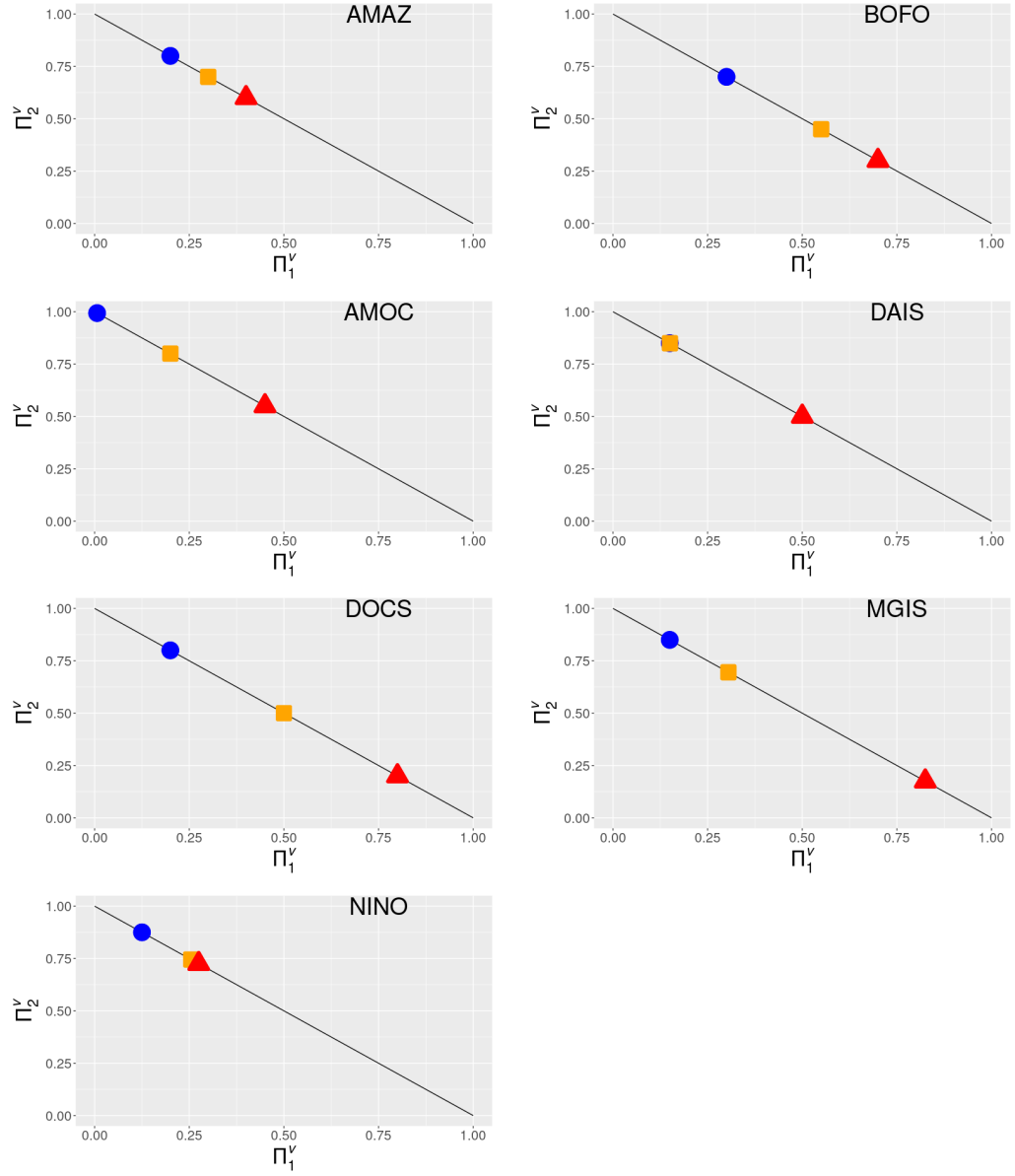
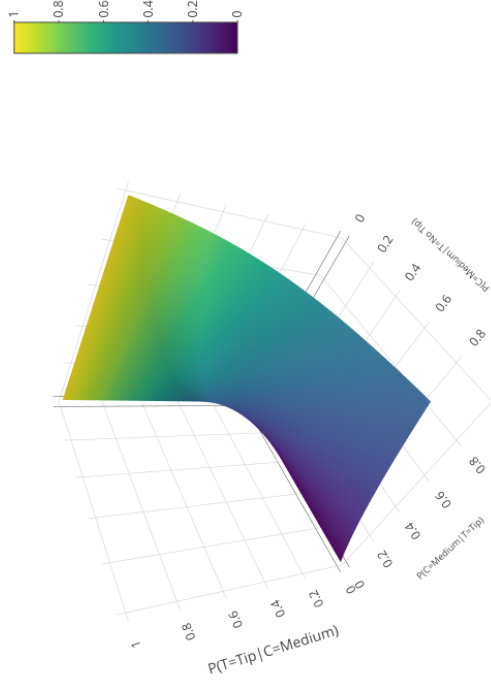
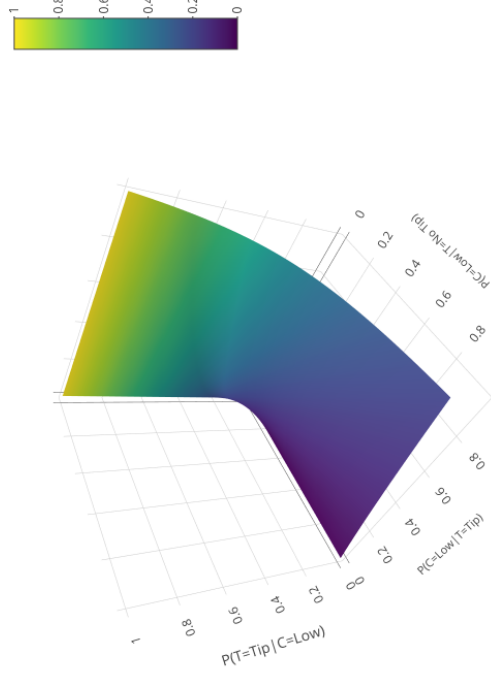


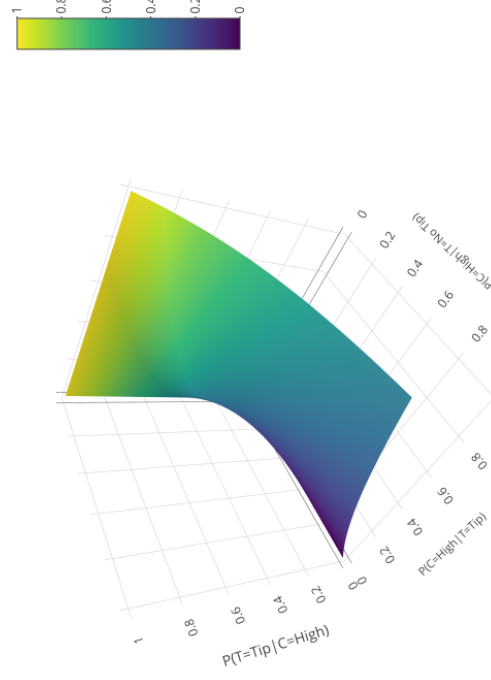
Fig. 1 Steiner points. The Low, Medium and High climate change scenarios are denoted by a circle, square and triangle respectively. Π_1^v is the probability that the tipping point is triggered, while Π_2^v is the probability that the tipping point is not triggered.



(a) Low temperature scenario.

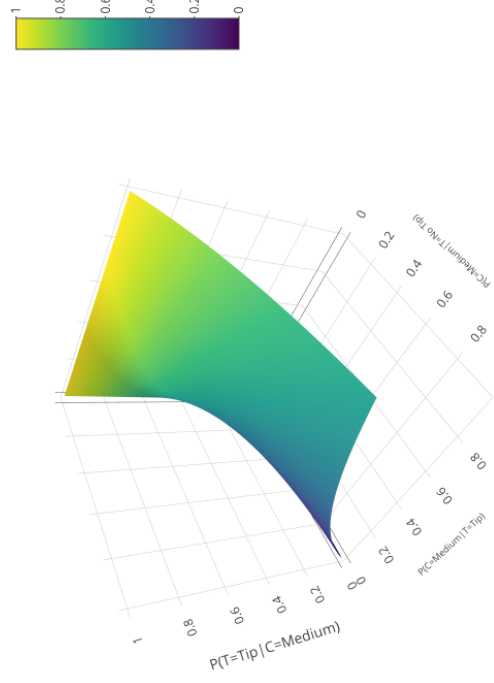


(b) Medium temperature scenario.

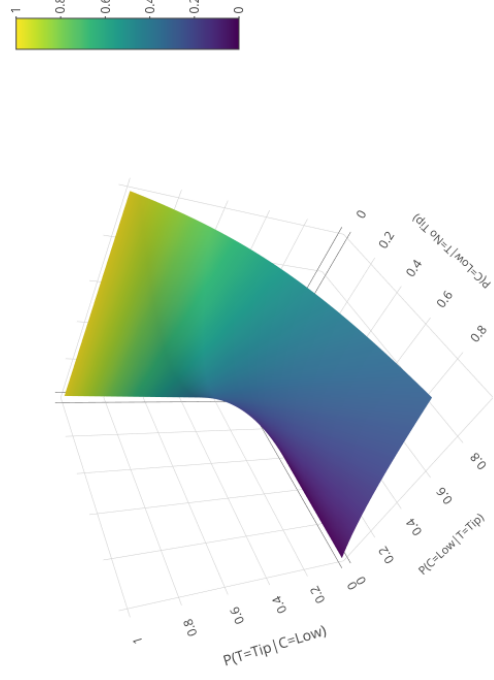


(c) High temperature scenario.

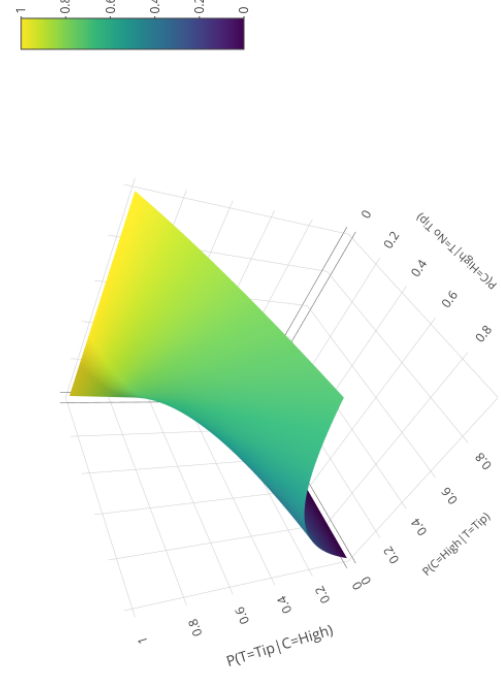
Fig. 2 Posterior surfaces for the AMAZ tipping point.



(b) Medium temperature scenario.

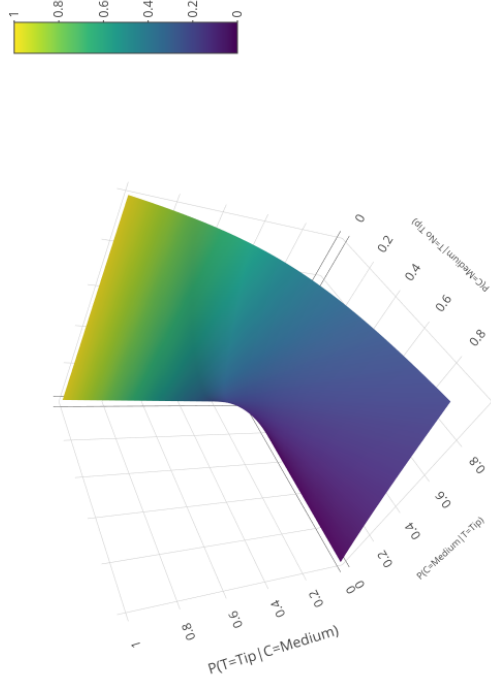


(a) Low temperature scenario.

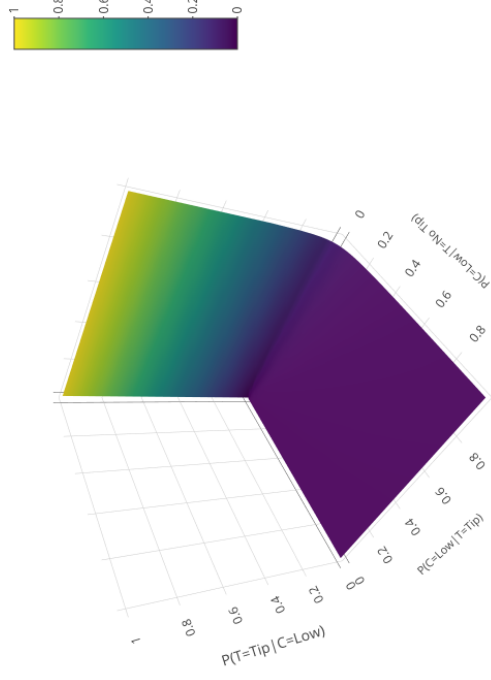


(c) High temperature scenario.

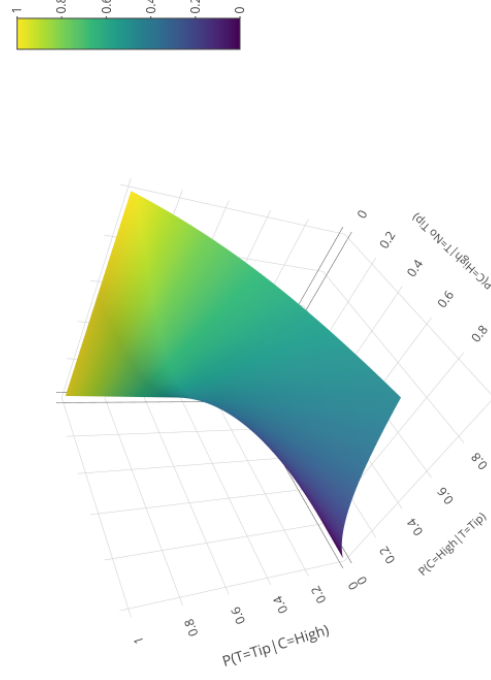
Fig. 3 Posterior surfaces for the BOFO tipping point.



(b) Medium temperature scenario.

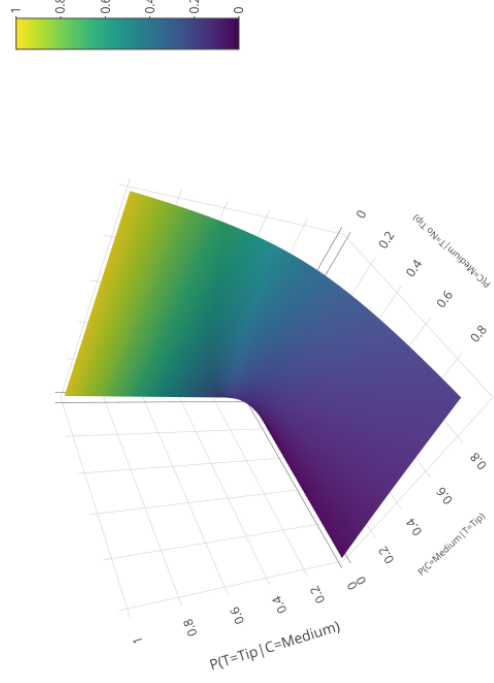


(a) Low temperature scenario.

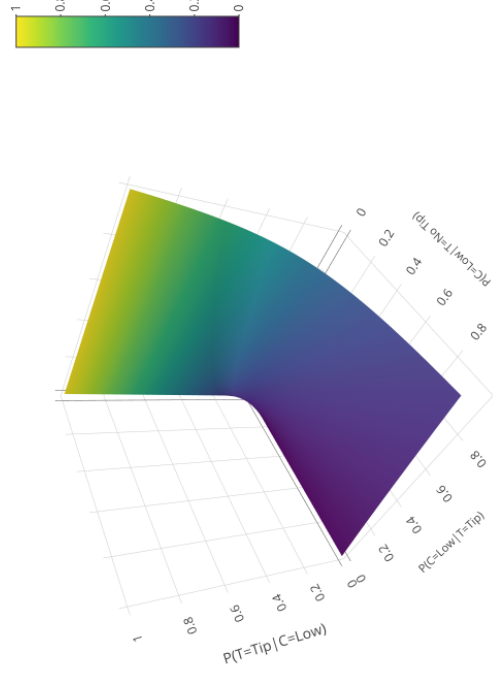


(c) High temperature scenario.

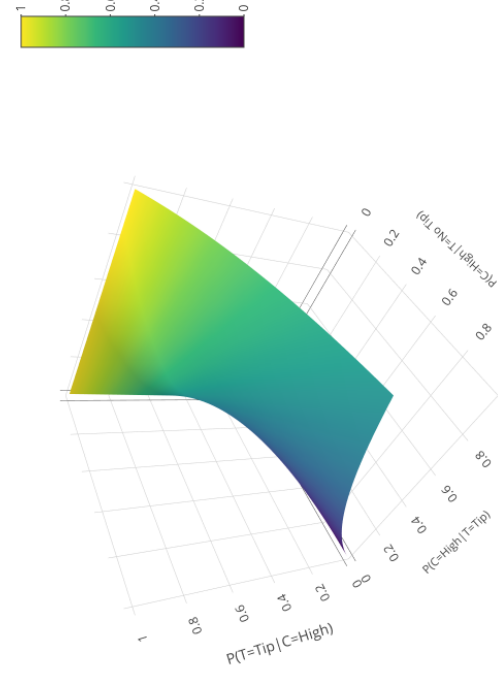
Fig. 4 Posterior surfaces for the AMOC tipping point.



(b) Medium temperature scenario.

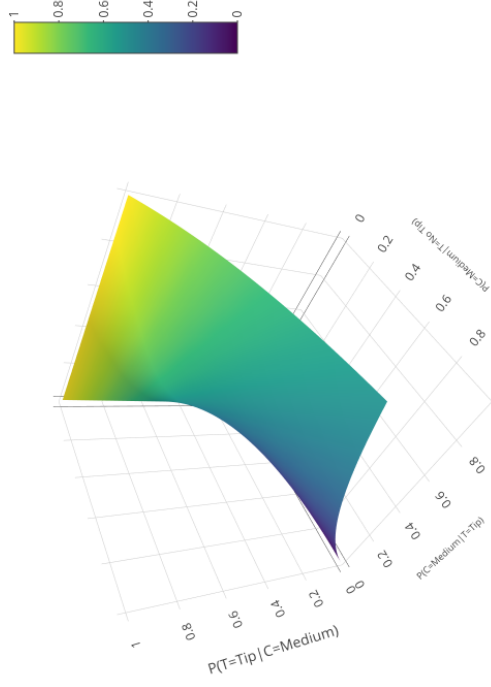


(a) Low temperature scenario.

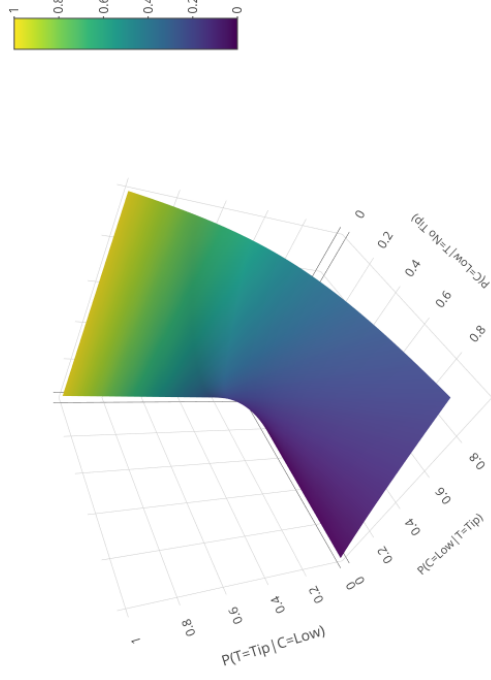


(c) High temperature scenario.

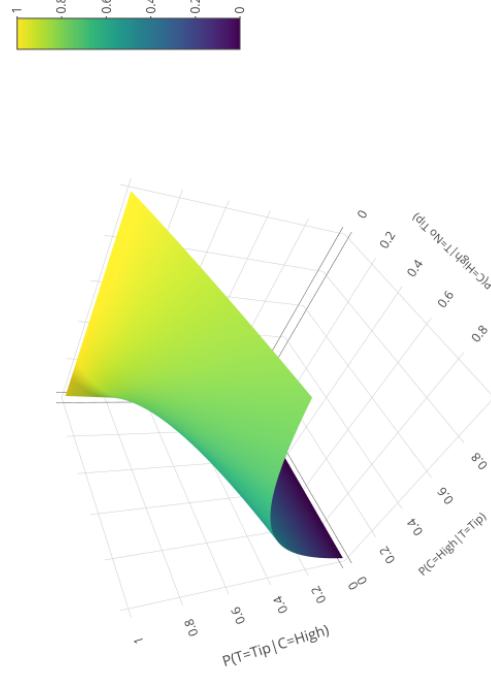
Fig. 5 Posterior surfaces for the DAIS tipping point.



(b) Medium temperature scenario.

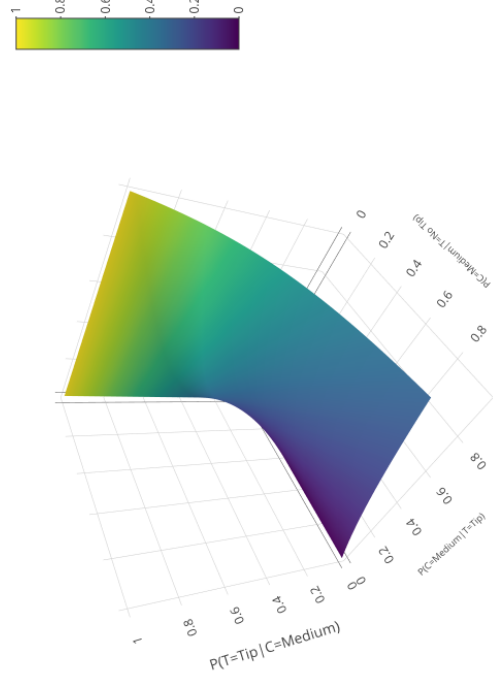


(a) Low temperature scenario.

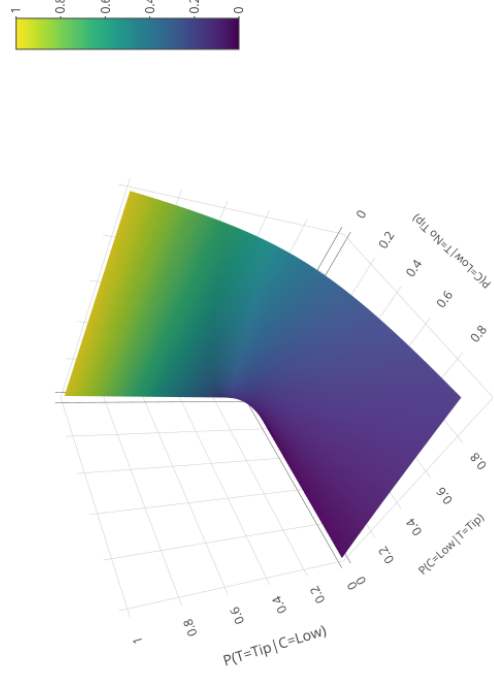


(c) High temperature scenario.

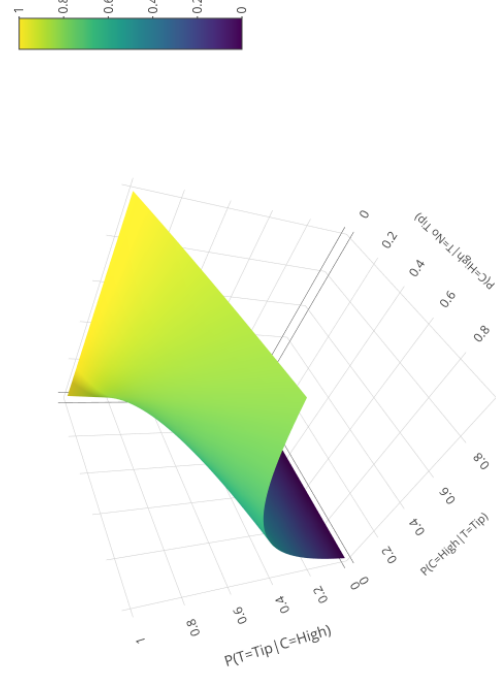
Fig. 6 Posterior surfaces for the DOCS tipping point



(b) Medium temperature scenario.

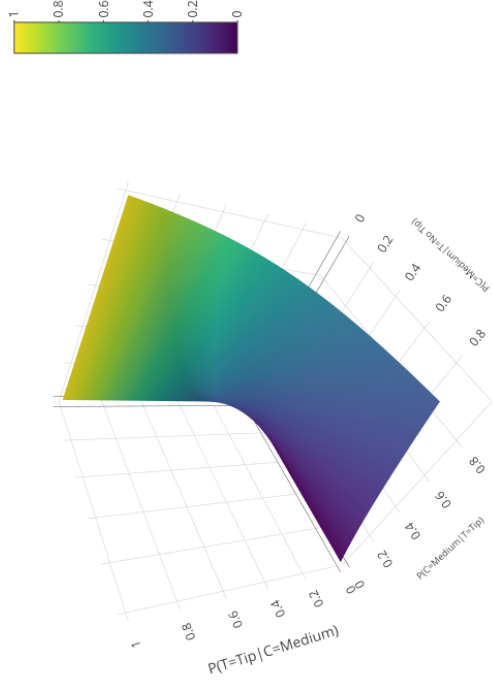


(a) Low temperature scenario.

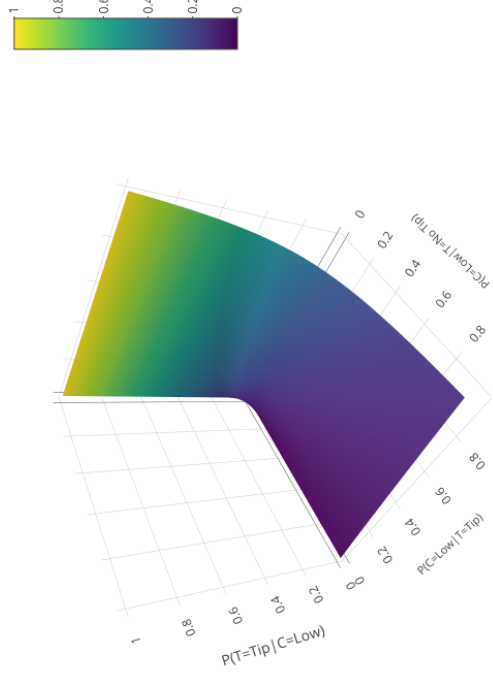


(c) High temperature scenario.

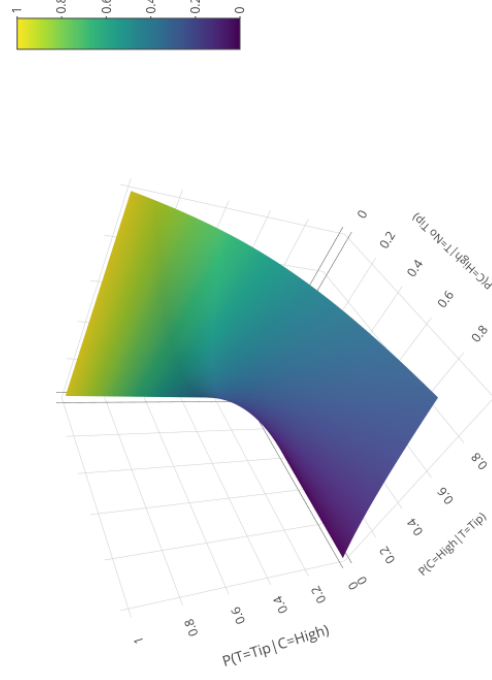
Fig. 7 Posterior surfaces for the MGIS tipping point.



(a) Low temperature scenario.



(b) Medium temperature scenario.



(c) High temperature scenario.

Fig. 8 Posterior surfaces for the NINO tipping point.

270 5 Conclusions

271 This paper considers the aggregation of experts' opinions about uncertain climate
272 change scenarios. The aggregation rule based on the Steiner point provides an
273 evaluation of the probability of occurrence of a tipping point.

274 In this context, the PM can evaluate the individual consistency (I-consistency)
275 of an expert, that is, whether an expert is able to provide coherent probabilistic
276 evaluations of possible future events, and the consistency of the whole group (G-
277 consistency), that is, whether the intersection of the probability sets associated with
278 the pool of experts is non empty. If the latter condition is verified the experts in the
279 pool share common opinions about future states of the world. The Steiner point allows
280 to automatically detect both individual and group consistency, but, more relevantly,
281 it can be updated by the Bayes rule, whenever new information is available, without
282 repeating a new trial of interviews among experts.

283 Ultimately, our results suggests that tipping points have higher probabilities to
284 occur under high temperature scenarios. If taken in isolation this result is perhaps
285 not surprising. A more interesting and concerning outcome is that the probability of
286 crossing a tipping point threshold is noticeably different from zero, even under lower
287 climate change scenarios and for the majority of tipping points under investigation.
288 The probability surfaces obtained via the Bayes rule show that committing to a low
289 climate change scenario may produce remote probabilities of realization of tipping
290 points, yet in most cases such probabilities tend to be larger than 10%. Significantly,
291 despite the fact that the data are over a decade old, to a large extent the results
292 obtained with our methodology overlap with those produced in [Armstrong McKay et](#)
293 [al. \(2022\)](#), even for the non occurrence of a tipping point for El Niño.

294 Due to limited data availability, we can only recover the marginal distribution of
295 tipping points. We update the resulting probabilities using a standard Bayes rule for
296 all the possible combinations of the likelihood of occurrence of a certain temperature

297 scenario. This approach generates posterior probability surfaces that may be inter-
298 preted as tipping points scenarios. The results suggest that some tipping points (the
299 dieback of the Boreal forest, the reorganization of the Atlantic meridional overturning
300 circulation, the decline of the ocean carbon sink and the melt of the West Antarctic
301 and Greenland ice) are extremely likely to occur under the high temperature scenario.
302 On the other hand, only committing to low climate change temperature corridors
303 would substantially reduce the probability of occurrence of most tipping points.

304 Our analysis, consistently with the recent literature on the topic, highlights the
305 fact that these type of events may turn out to be one of the main climatic emergencies
306 in the upcoming years. Future research can further contribute to the understanding
307 of these issues by addressing problems concerning elicitation data, the construction of
308 joint distributions for tipping events and the estimation of causal relationships among
309 tipping points.

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456

457 **Appendix A Figures**

458 This section displays a set of dumbbell plots that describe the data used in the analysis.

459 The plots are similar to those in Figure 1 of [Kriegler et al. \(2009\)](#). Furthermore, in

460 [Kriegler et al.](#) some of the experts are recognized as *core experts*. The caption of each

461 plot indicates which experts are not core experts.

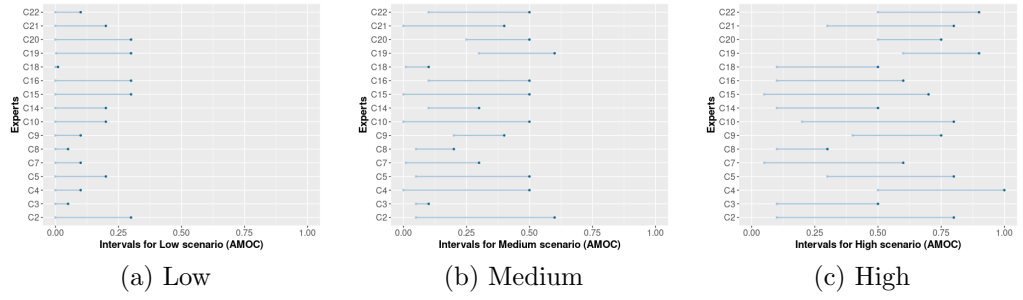


Fig. A1 Elicited probability intervals for the AMOC tipping point. In [Kriegler et al. \(2009\)](#) C9, C20 and C22 are not classified as core experts.

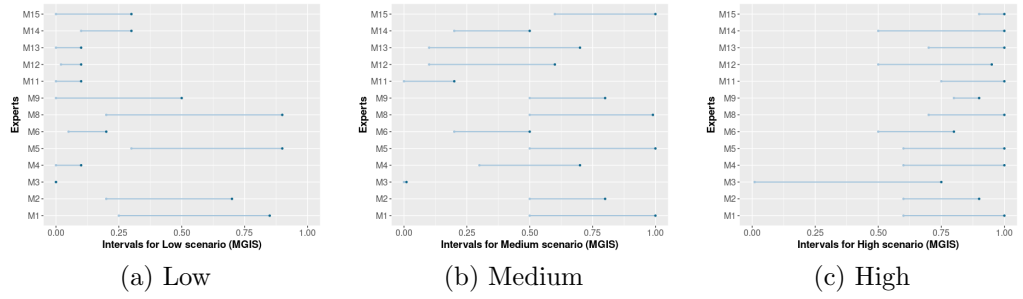


Fig. A2 Elicited probability intervals for the MGIS tipping point. In [Kriegler et al. \(2009\)](#) M1, M3 and M14 are not classified as core experts.

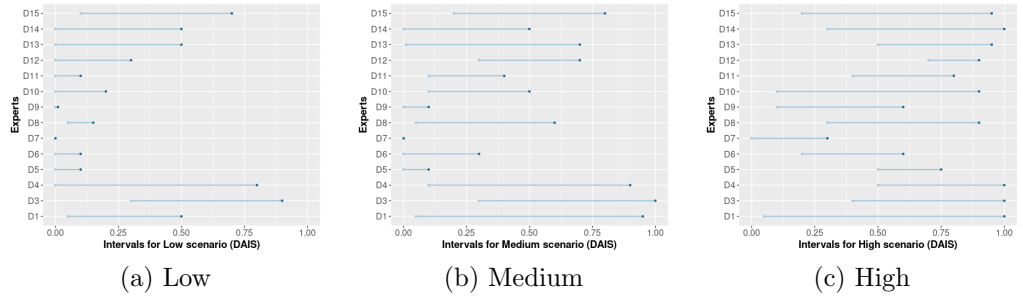


Fig. A3 Elicited probability intervals for the DAIS tipping point. In [Kriegler et al. \(2009\)](#) D7 and D10 are not classified as core experts.

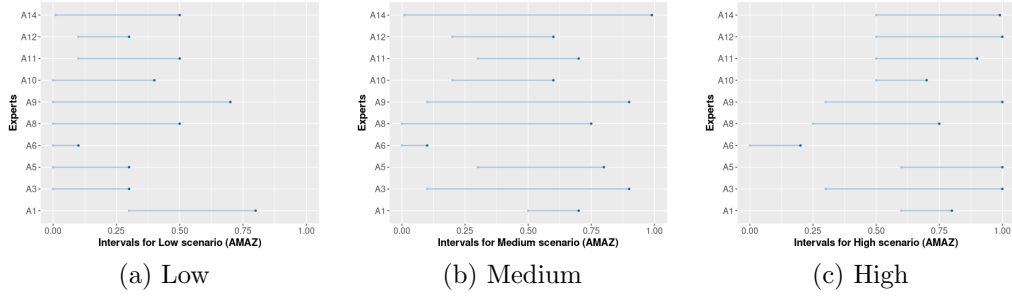


Fig. A4 Elicited probability intervals for the AMAZ tipping point. In [Kriegler et al. \(2009\)](#) A1, A6, A10 and A12 are not classified as core experts.

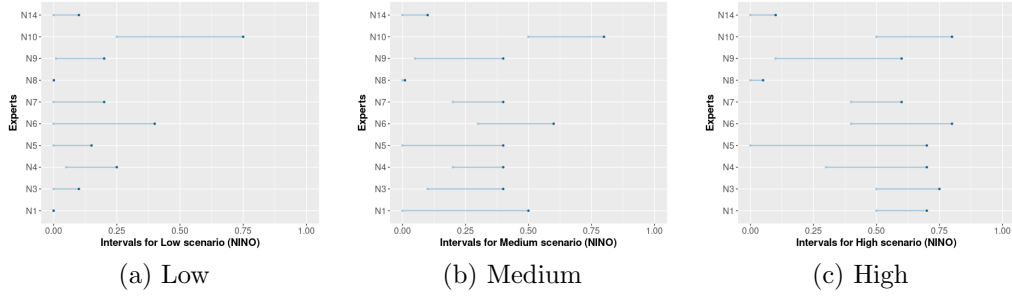


Fig. A5 Elicited probability intervals for the NINO tipping point. In [Kriegler et al. \(2009\)](#) N3, N6 and N10 are not classified as core experts.

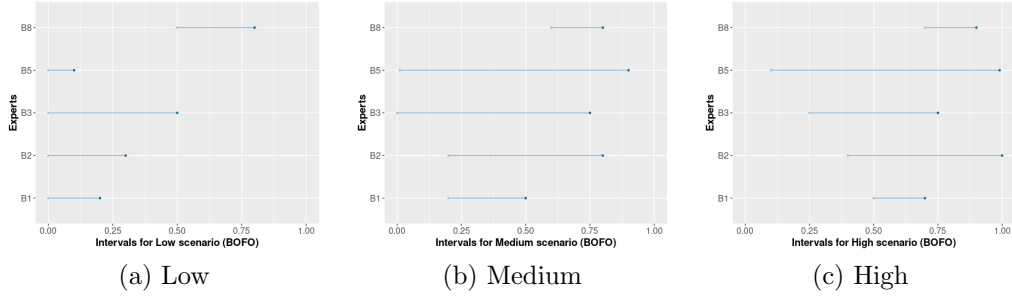


Fig. A6 Elicited probability intervals for the BOFO tipping point. In [Kriegler et al. \(2009\)](#) B1 is not classified as a core experts.

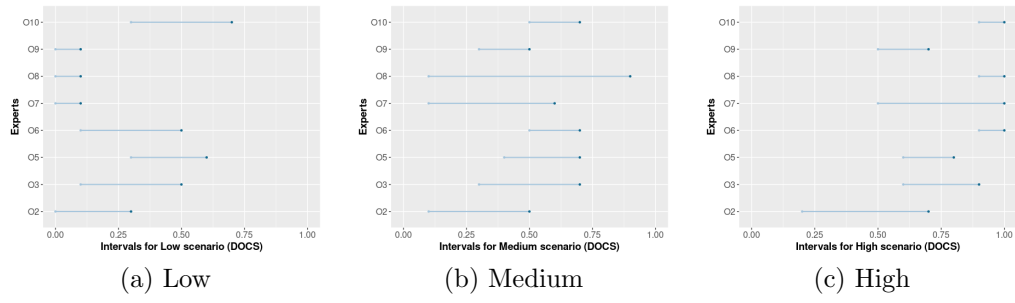


Fig. A7 Elicited probability intervals for the DOCS tipping point. In [Kriegler et al. \(2009\)](#) O2 is not classified as a core experts.